TPP ECONOMY REVIEW

Technology and Policy Program

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ECONOMY*

I. **Production**: Cost Functions and Profit Maximization
II. **Consumption**: Benefit and Demand Functions
III. **Market**: Supply and Demand
IV. Government Interventions

** This material builds on previous content created by Jesse Jenkins and Mark Staples*

I - Production

PRODUCTION COST FUNCTIONS

TOTAL PRODUCTION COST FUNCTION

Q units

$$C(Q) = P$$

cost in $ to produce Q units

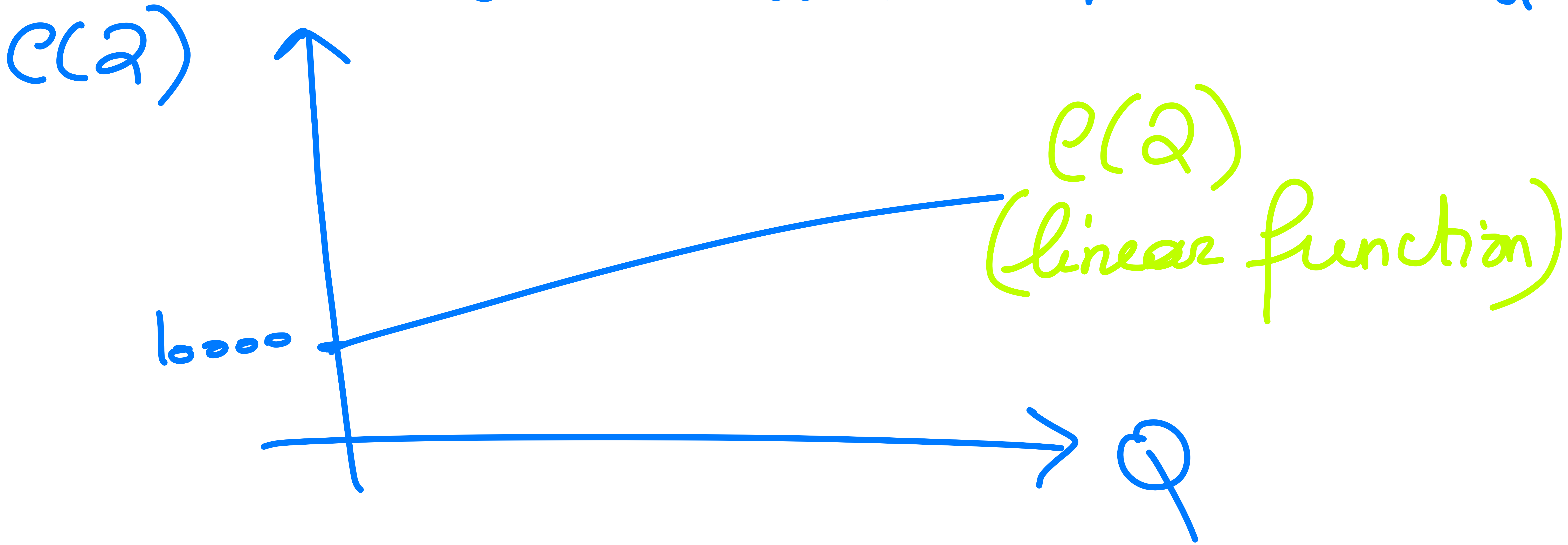
SIMPLIFIED BAKERY EXAMPLE

- It costs a bakery \$10,000 to buy a new oven. Each loaf requires \$5 of ingredients, and the baker is paid \$10 per loaves produced. What is the **production cost function**?

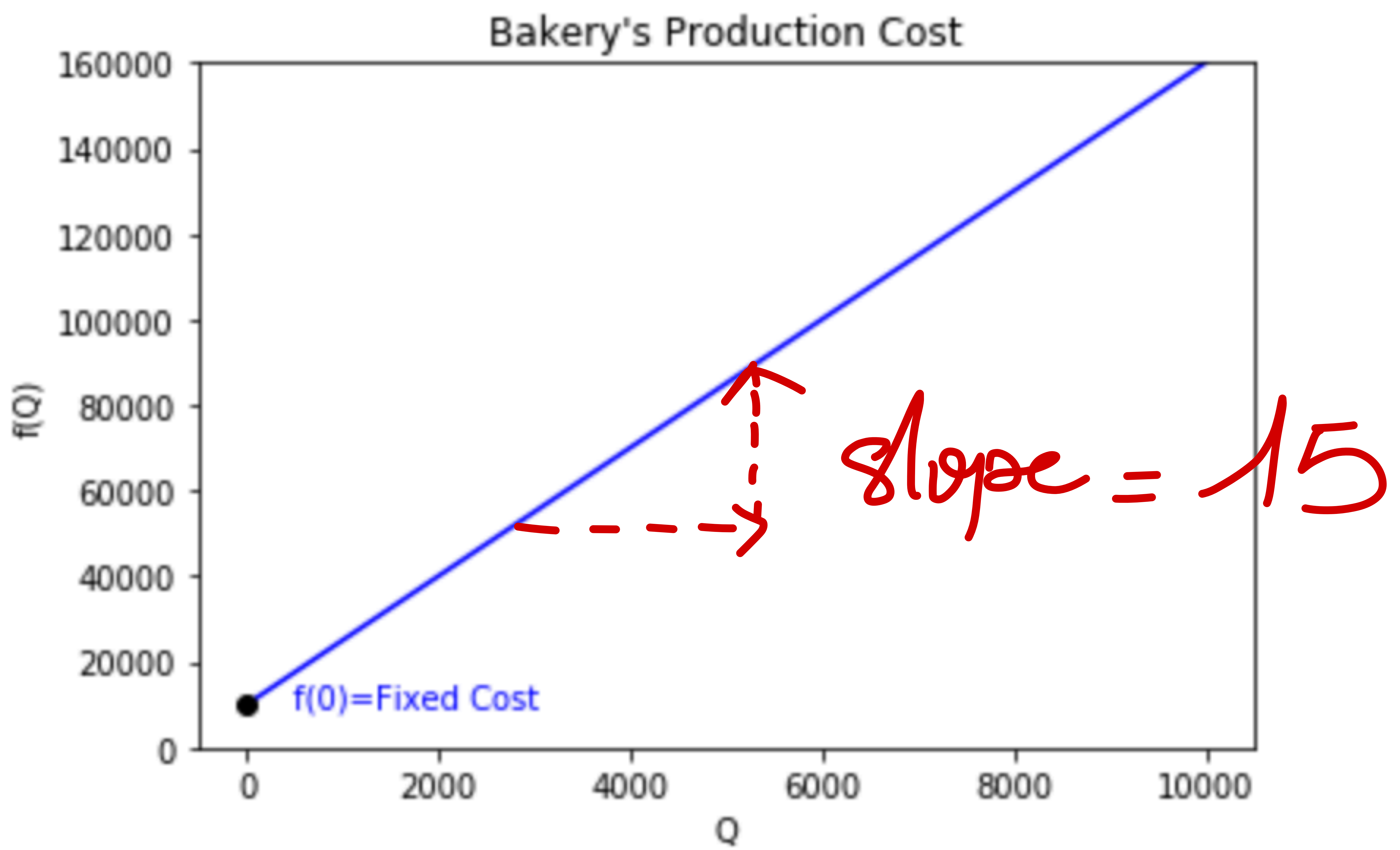
$$C(Q) = \underbrace{10000}_{\text{fixed cost}} + \underbrace{(5+10)}_{\text{variable cost}} Q$$

FIXED AND VARIABLE COSTS

- Does the oven's price vary with the quantity of loaves produced? NO → long term cost
- Does the ingredients and labor's vary with the quantity of loaves produced? YES → unit-dependent cost



FIXED AND VARIABLE COSTS



- How much does it cost to produce an extra loaf (a.k.a 'production unit'?)

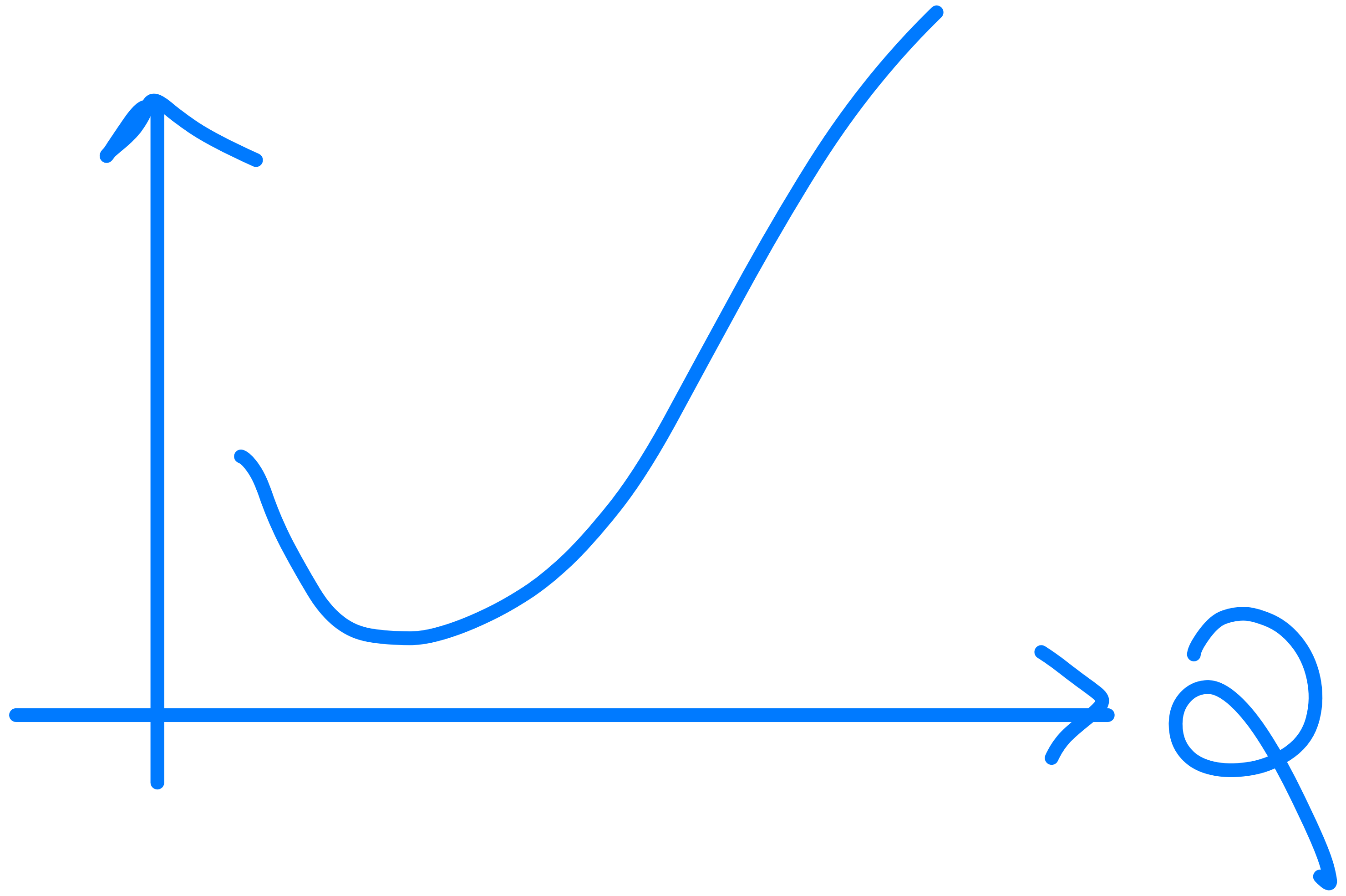
MARGINAL COST FUNCTION

$/unit

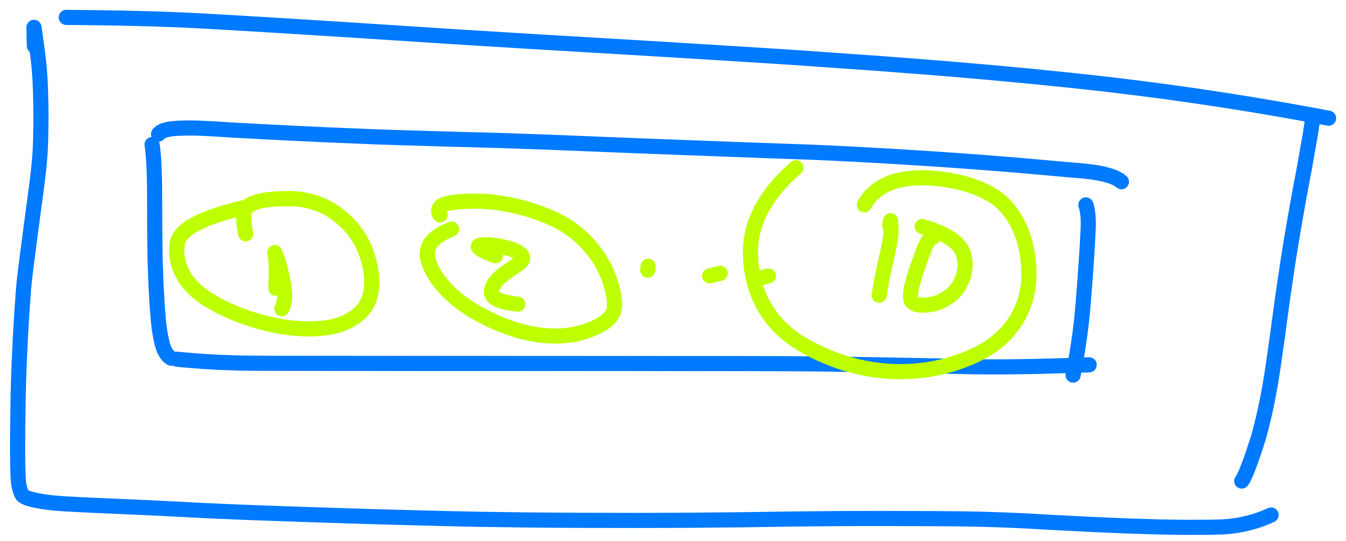
$$MC(Q) = C'(Q)$$

cost for 1 extra unit

MC(Q)



MARGINAL COST FUNCTION'S SHAPE EXPLAINED



prepare the paste (10 min)

2nd cookie : 15 sec

6th cookie : 10 min (new paste)

11th cookie : 1h! cooking time

- ➤ You are baking cookies in a oven's rack that can fit 10 cookies.
- ➤ It takes you 5min to bake the first cookie. Preparing the second cookies takes you an additional 15sec.
- ➤ How much more time do you need for the third cookie?
- ➤ And for the eleventh?

RETURN TO SCALE

the more units you produce, the lower the marginal cost

$$C''(Q) = MC'(Q)$$

- Increasing Return To Scale: $MC'(Q) \searrow \Leftrightarrow C''(Q) < 0$
- Decreasing Return To Scale: vice versa
- Constant Return To Scale: $MC'(Q) = \text{cste} \Leftrightarrow C''(Q) = 0$

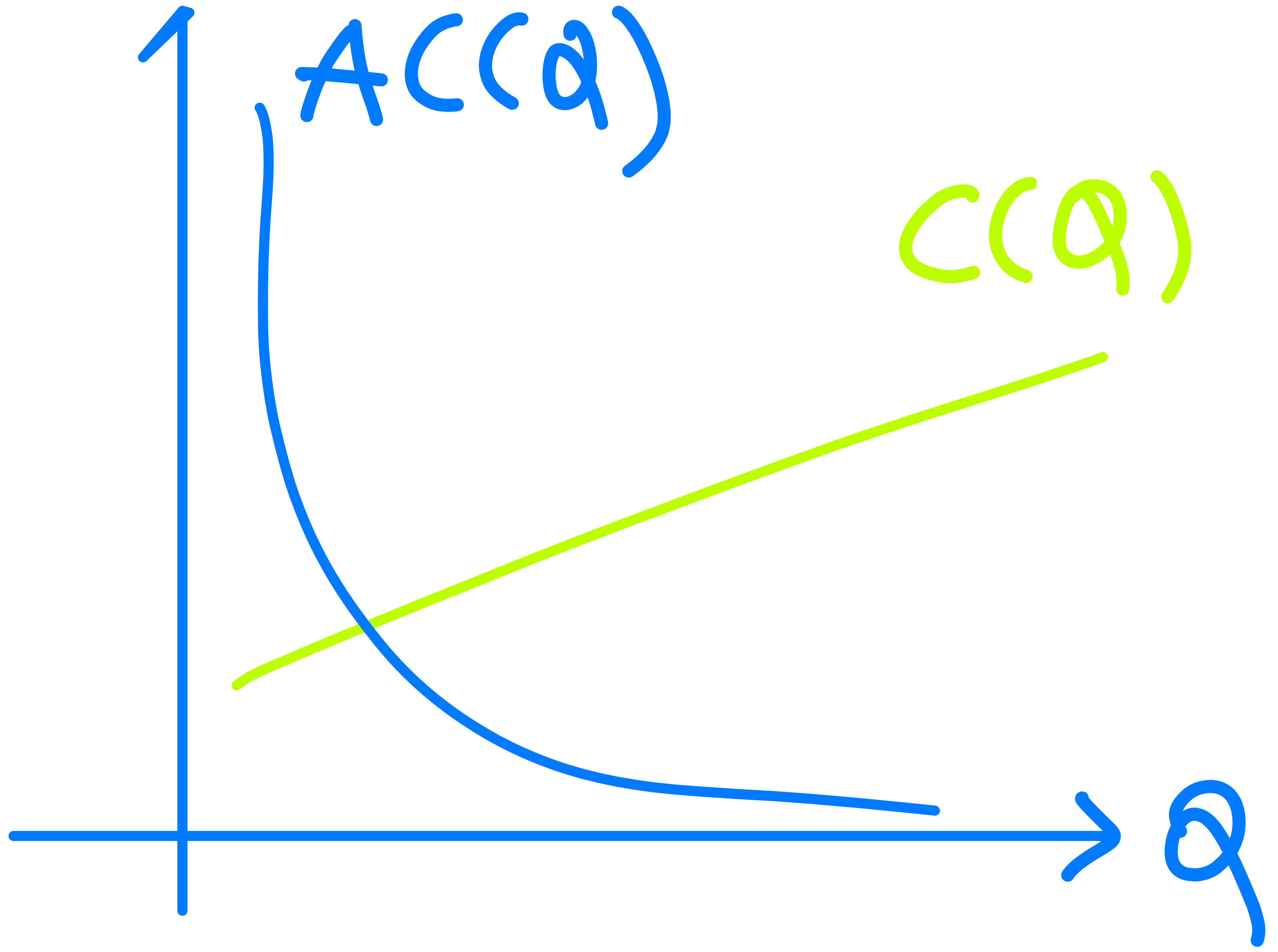
- How would you qualify the bakery's return to scale? → constant

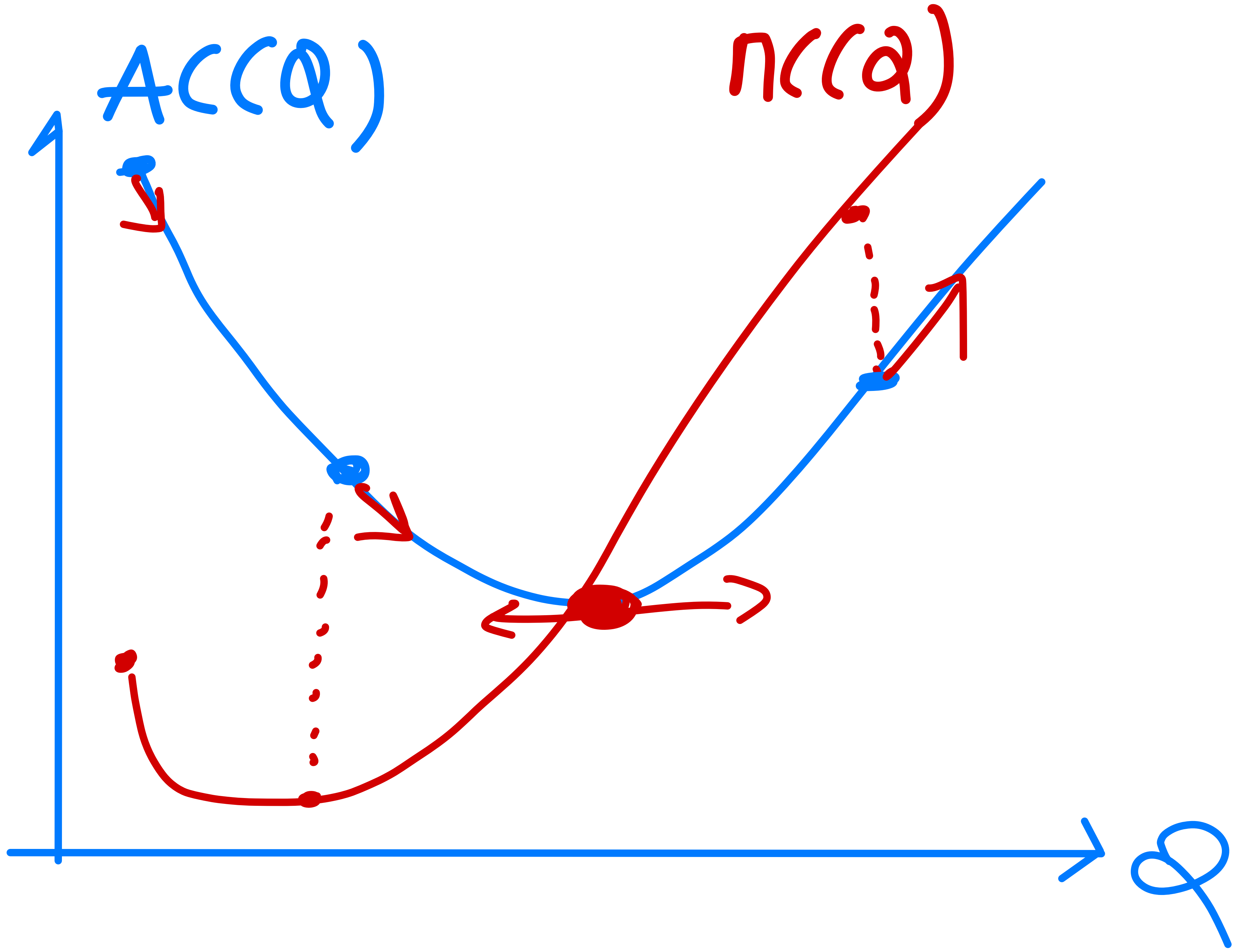
increasing then decreasing ← The cookies baker's? Give an example of decreasing/ → $C(Q) = Q^2$ increasing return to scale. → $C(Q) = \sqrt{Q}$

AVERAGE PRODUCTION COST

$$AC(Q) = \frac{C(Q)}{Q}$$

AVERAGE PRODUCTION COST SHAPE





DIFFERENCE BETWEEN MARGINAL COST AND AVERAGE COST*

- Your **average** before the final exam in your class is 85.
- If you get 80 at the final, your *marginal improvement* is below your average and your **average** goes down.
- If you get 90 at the final, your *marginal improvement* is above your average and your **average** goes up.

* *Example from thoughtco.com*

DIFFERENCE BETWEEN MARGINAL COST AND AVERAGE COST

- Formally, observe that $MC(Q) = C'(Q)$ and
$$AC(Q) = \frac{C(Q)}{Q}$$ → what is $AC(Q)$ monotonicity?

find Q s.t. $AC'(Q) \geq 0$; ≤ 0

- Note that
$$AC'(Q) = \frac{f'(Q)Q - C(Q)}{Q^2} = \frac{MC(Q) - AC(Q)}{Q}$$ => $AC'(Q) \geq 0$ i.i.f $MC(Q) \geq AC(Q)$

- So the average cost decreases as long as the marginal cost is below the average cost, and vice versa. Both curves cross when the average cost is minimal.

ECONOMY OF SCALE

- When $AC(Q)$ the average cost declines as a function of the quantity produced, we talk about **economies of scale.**

- Economy of scale: $AC'(Q) < 0$
- Diseconomy of scale: $AC'(Q) > 0$

- Did the bakery experience economy of scale?

$AC(Q) = 10000/Q + 15$; $AC'(Q) = \frac{-10000}{Q^2} < 0$ YES!

PROFIT MAXIMIZATION

NET PROFIT OR PRODUCER SURPLUS

$$\Pi(Q) = R(Q) - C(Q)$$

revenue

(at constant selling price, $R(Q) = P(Q)$)

REVENUE OF A PRICE-TAKING FIRM

- What would it mean for a company to have the following revenue function (P is fixed)?

$$R(Q) = PQ$$

producer does not impact the price

- Which kind of companies would have such a revenue function?

MAXIMIZE REVENUE OF A PRICE-TAKING FIRM

- $\max_Q \Pi(Q) = \max_Q \left(R(Q) - C(Q)\right) = \max_Q \left(PQ - C(Q)\right)$
- $\Pi'(Q_0) = P - C'(Q_0) = P - MC(Q_0) = 0 \Rightarrow MC(Q_0) = P$
- If you can earn \$100 for each unit sold, you earn money on each unit sold such that the marginal cost to produce the unit is below \$100.

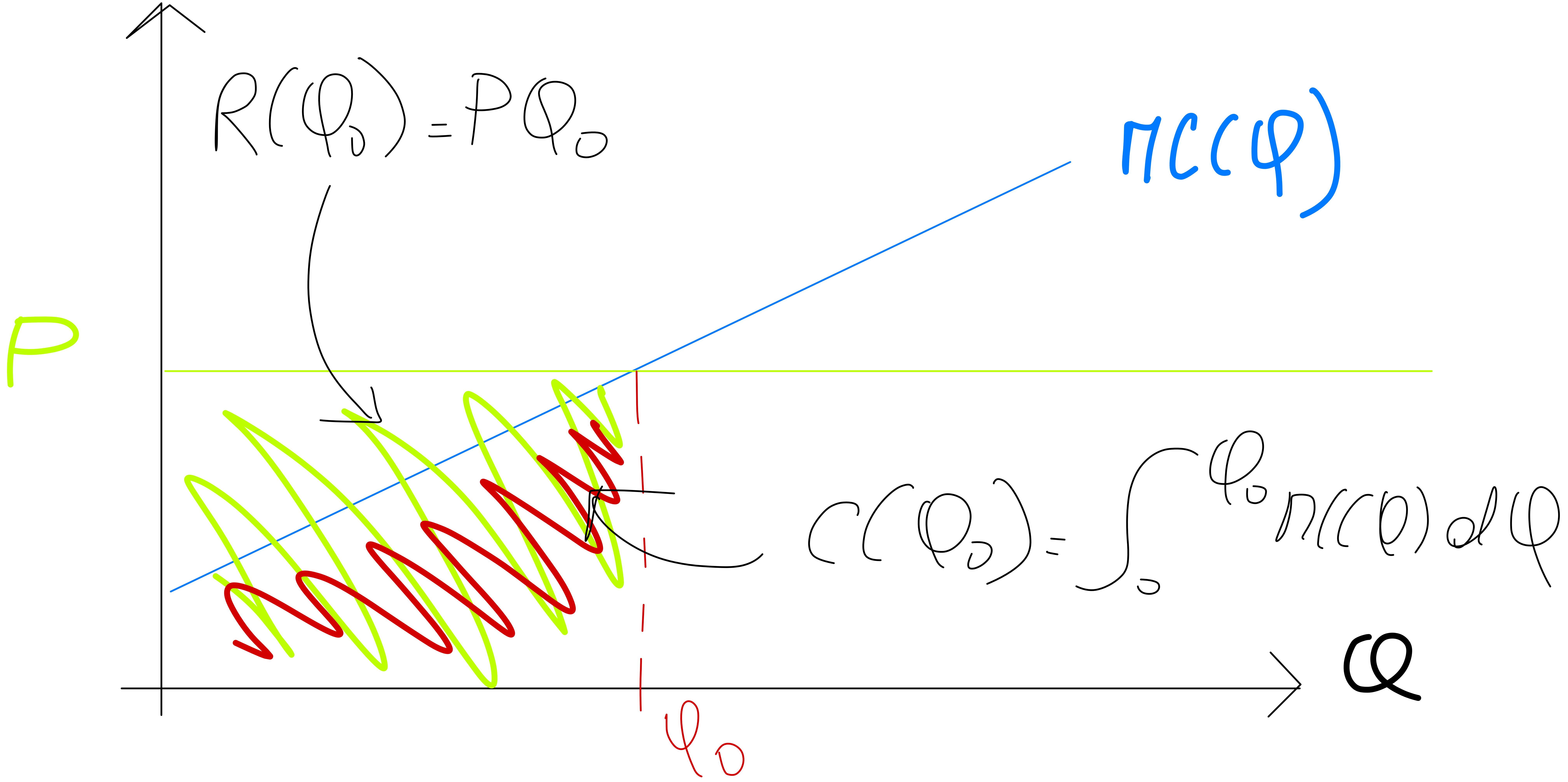
MAXIMIZE REVENUE OF A PRICE-TAKING FIRM

- If $C(Q) = 1000 + 100Q + 3Q^2$ for a price-taking firm with a market price of \$400, how many units shall be produced to maximize profit? What is the producer surplus then?

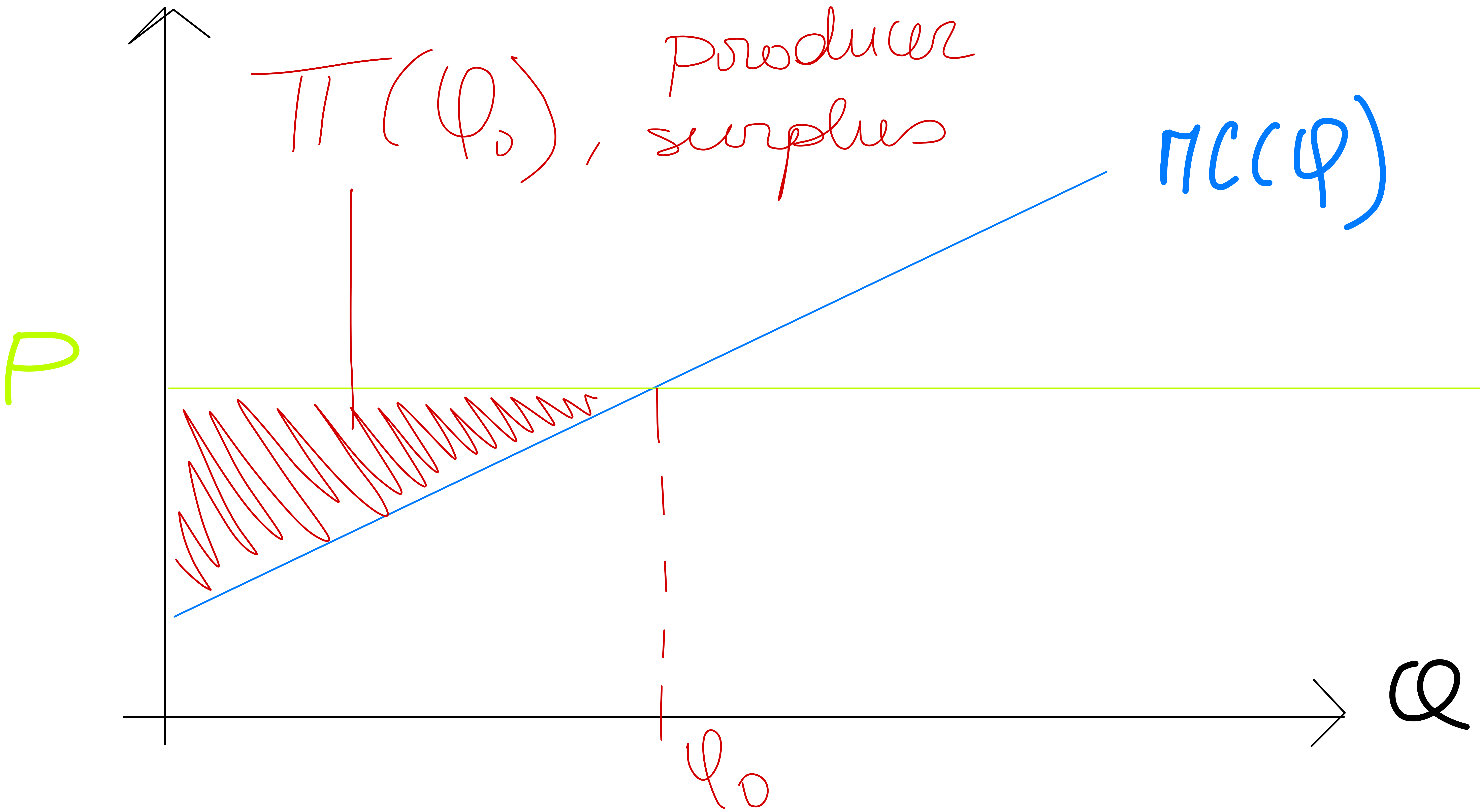
$$MC(Q) = 100 + 6Q = 400$$

$$\Rightarrow Q = 50$$

COST, MARGINAL COST AND PROFIT VISUALLY



COST, MARGINAL COST AND PROFIT VISUALLY



II - Consumption

BENEFIT FUNCTIONS

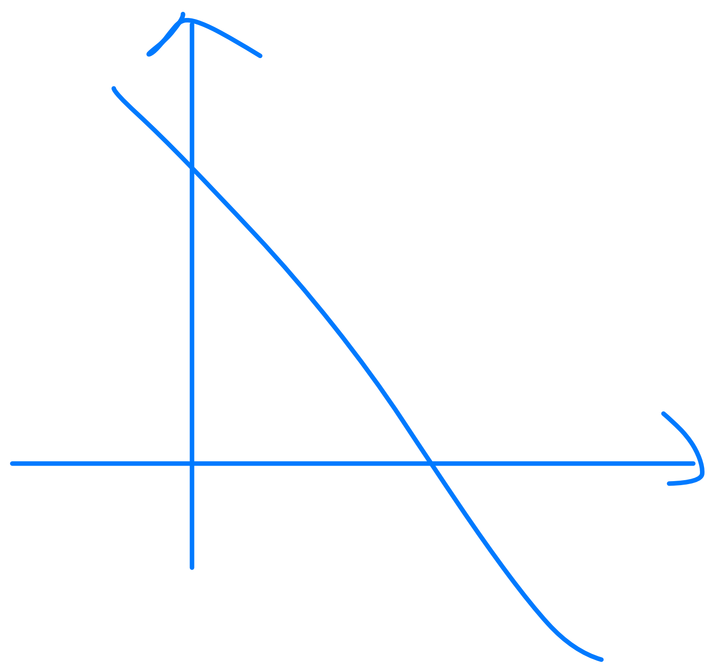
MARGINAL BENEFIT FUNCTIONS

$$MB(Q) = P$$

price one
is willing to
pay for Q units

EXAMPLES

- $MB_1(Q) = 5 - Q$
- $MB_2(Q) = 2 - 2Q/3$
- Why is the function decreasing?
- What is it reminiscent of?



can't eat too many Snickers® in a row

TOTAL BENEFIT

$$B(Q) = \int_0^Q MB(q)dq$$

NET BENEFIT OR CONSUMER SURPLUS

$$\zeta(Q) = B(Q) - PQ$$

in

customer

don't influence
market price

MAXIMIZE BENEFIT OF A CONSUMER

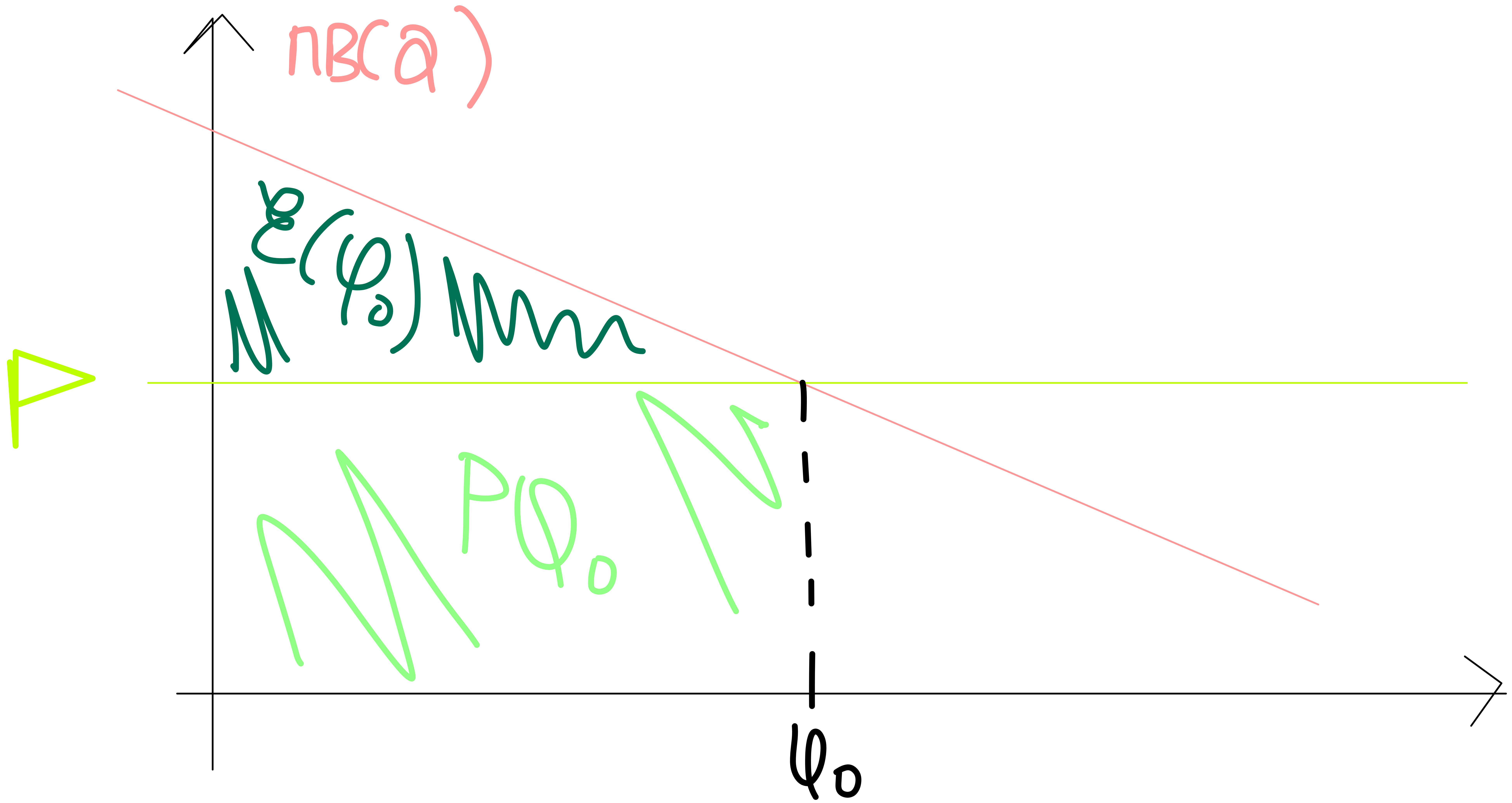
$$\max_{Q} \xi(Q) \Leftrightarrow$$

find Q_0 s.t. $B'(Q_0) - P = 0$

$\Leftrightarrow \quad MB(Q_0) = P$

Intuition ?

MARGINAL BENEFIT, BENEFIT AND NET BENEFIT VISUALLY



III - Markets

COST AND BENEFIT FUNCTIONS

- Cost Function
- Benefit Function

$/unit

- $MC(Q) = P$
- $MB(Q) = P$ and $D(P) = Q$

marginal change

$

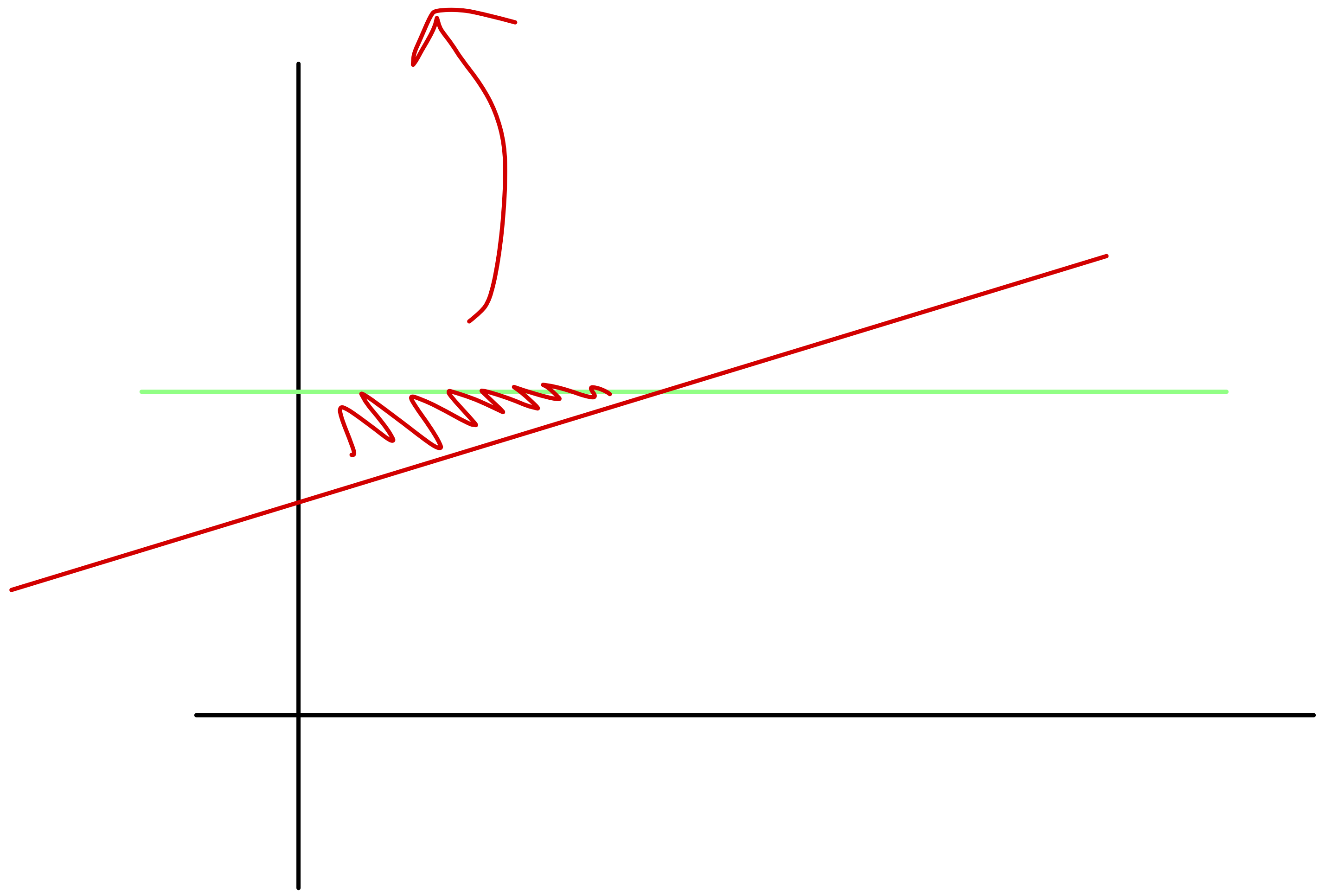
- $C(Q) = P$
- $B(Q) = P$

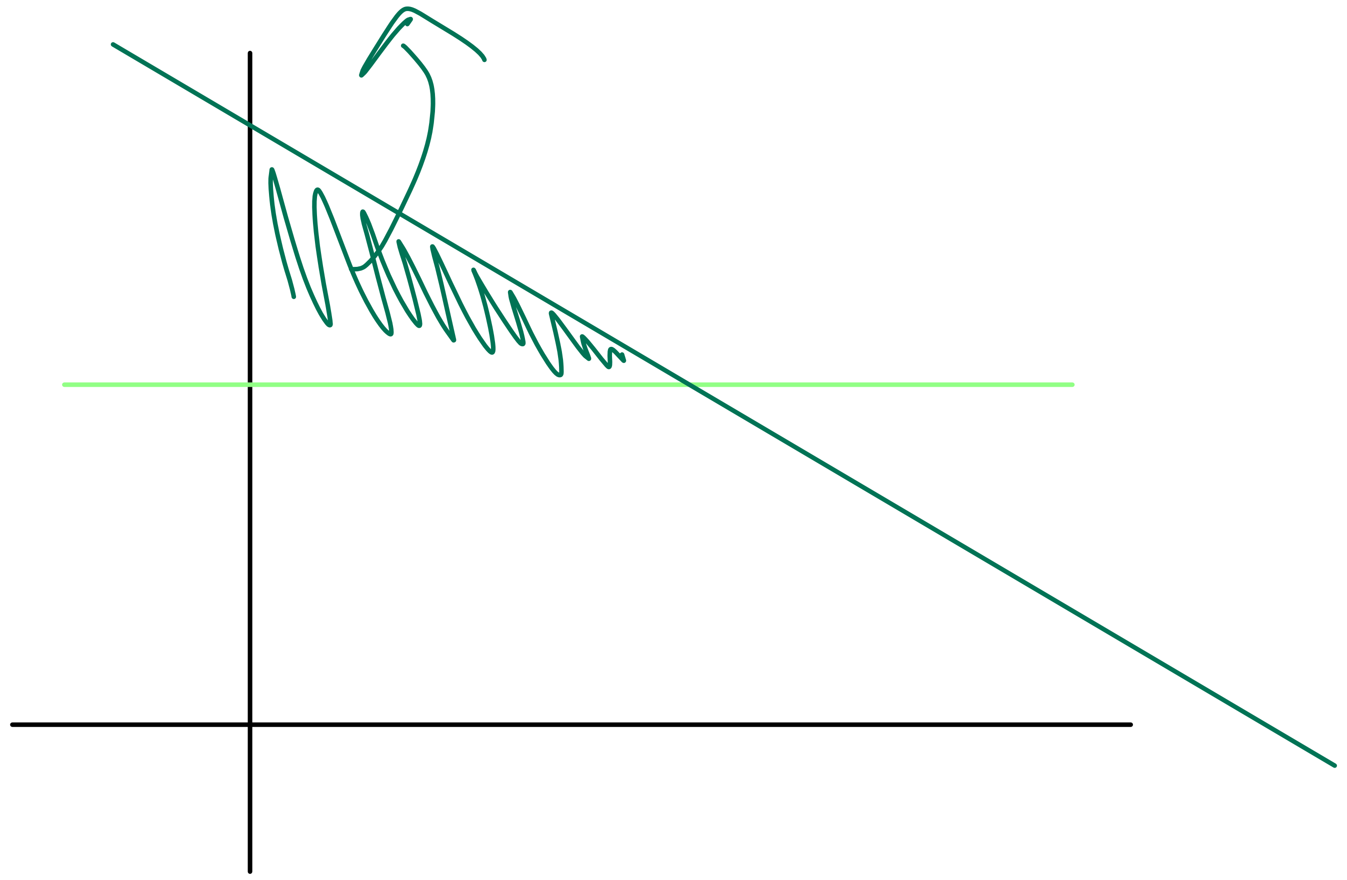
total "outcome"

$

- $\Pi(Q) = R(Q) - C(Q)$
- $\zeta(Q) = B(Q) - PQ$

"profit"





SUPPLY AND DEMAND CURVES

- Supply Curve: aggregate of all marginal cost functions for all producers

$$P = S(Q) = MC_{agg}(Q)$$

- Demand Curve: aggregate of all marginal benefit cost functions for all consumers

$$P = D(Q) = MB_{agg}(Q)$$

- What shall be the monotonicity of the supply and demand curves?

it costs more to produce more
one is less willing to consume more units

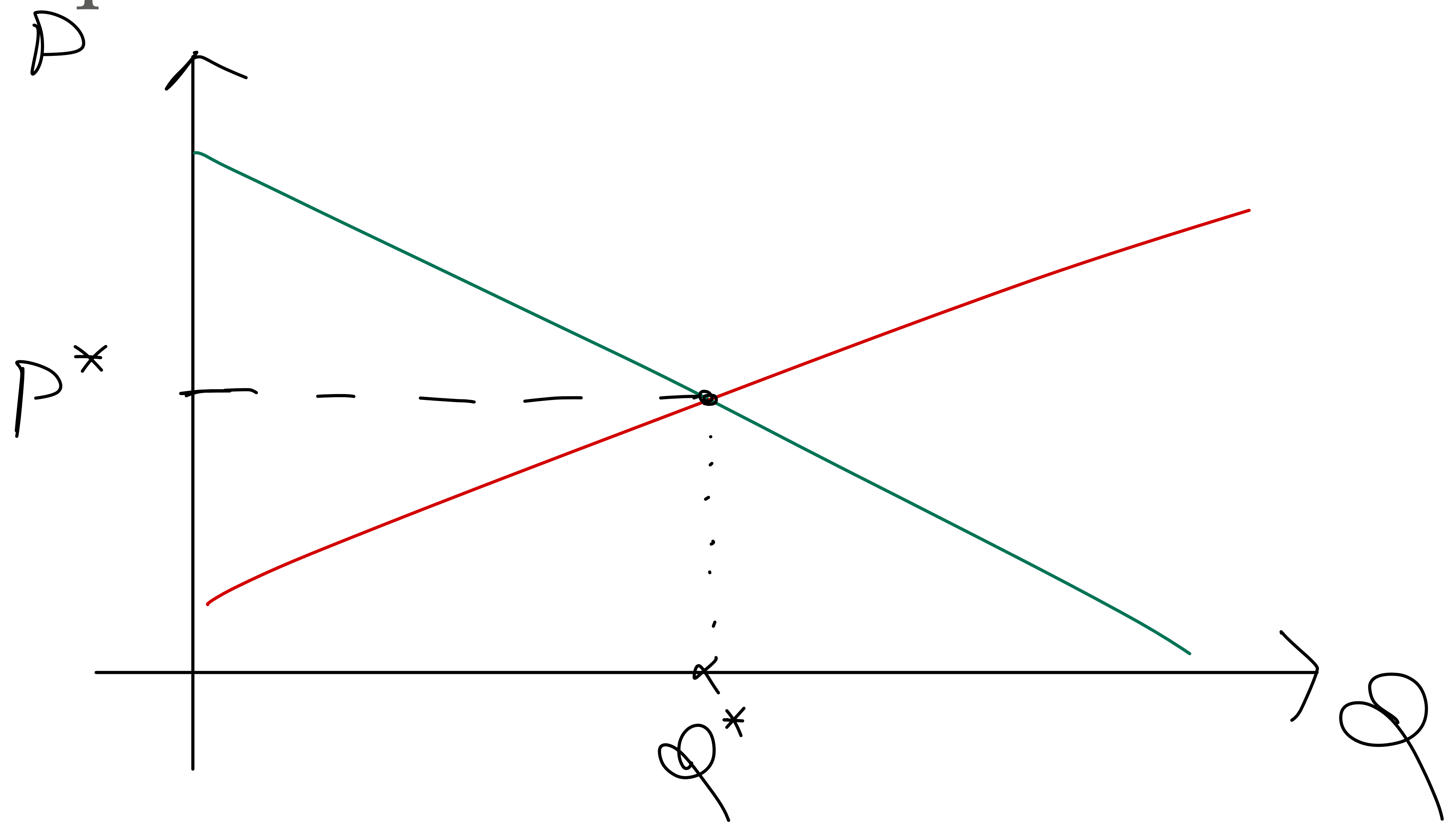
MARKET EQUILIBRIUM

MARKET EQUILIBRIUM

- The market is at equilibrium when:

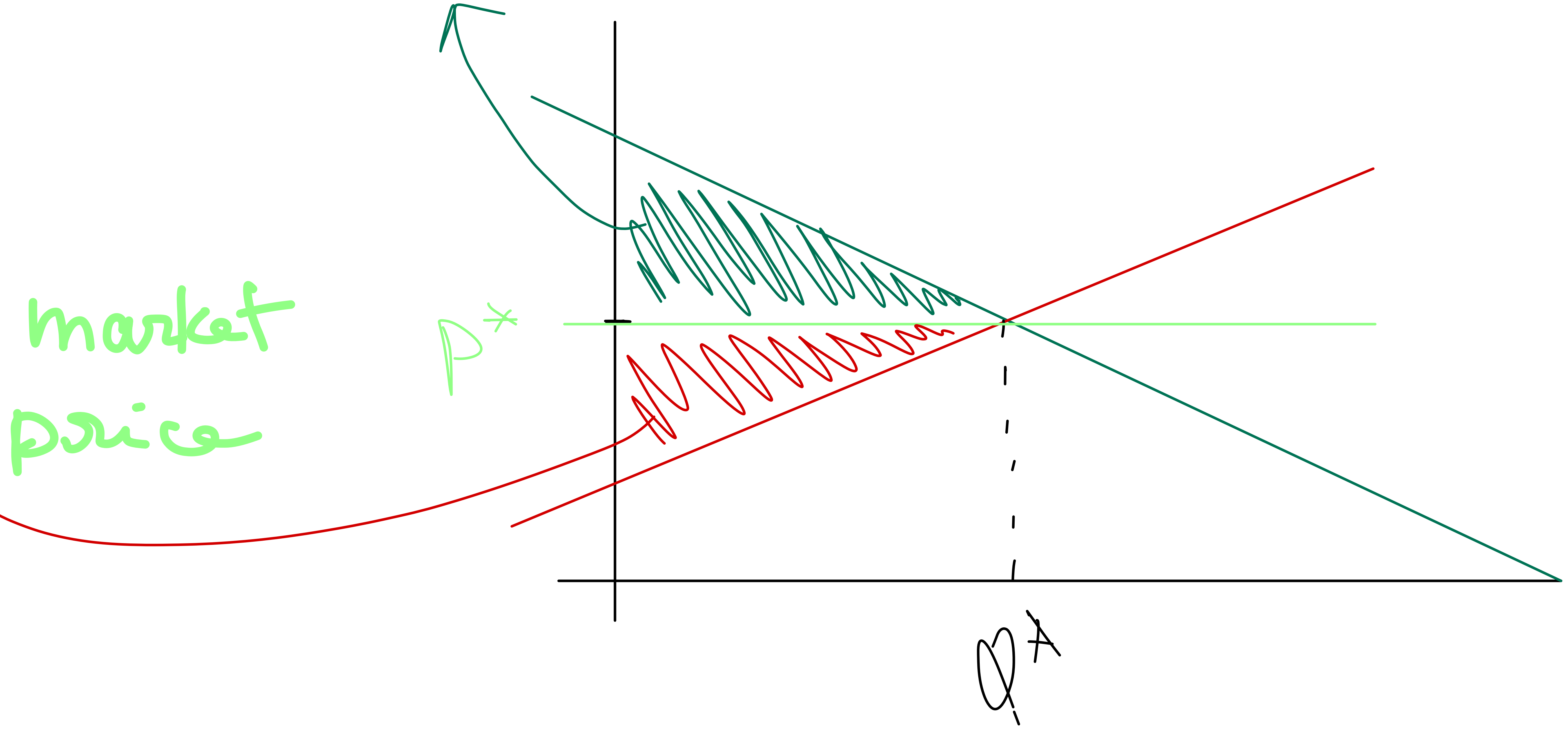
$$P^* = S(Q^*) = D(Q^*)$$

- P^* is the ***market clearing price***, the price at which supply equals demand so the market is *clear*.



CONSUMER AND PRODUCER SURPLUS VISUALLY

- $\Pi(Q) = R(Q) - C(Q) = \int P - MC(Q)$
- $\zeta(Q) = B(Q) - PQ = \int MC(Q) - P$



MARKET EQUILIBRIUM — EXAMPLE

- Let a market in which the aggregate cost function is $C(Q) = 25 + 20Q + 3Q^2$ and the aggregate demand function is $D(Q) = 200 - 3Q$.
- What is the market equilibrium?
- How much profit do producers make?
- How much net benefit consumers enjoy?

$MC(Q) = 20 + 6Q$

$MB(Q) = 200 - 3Q$

$20 + 6Q^* = 200 - 3Q^*$

$\Leftrightarrow Q^* = 20$

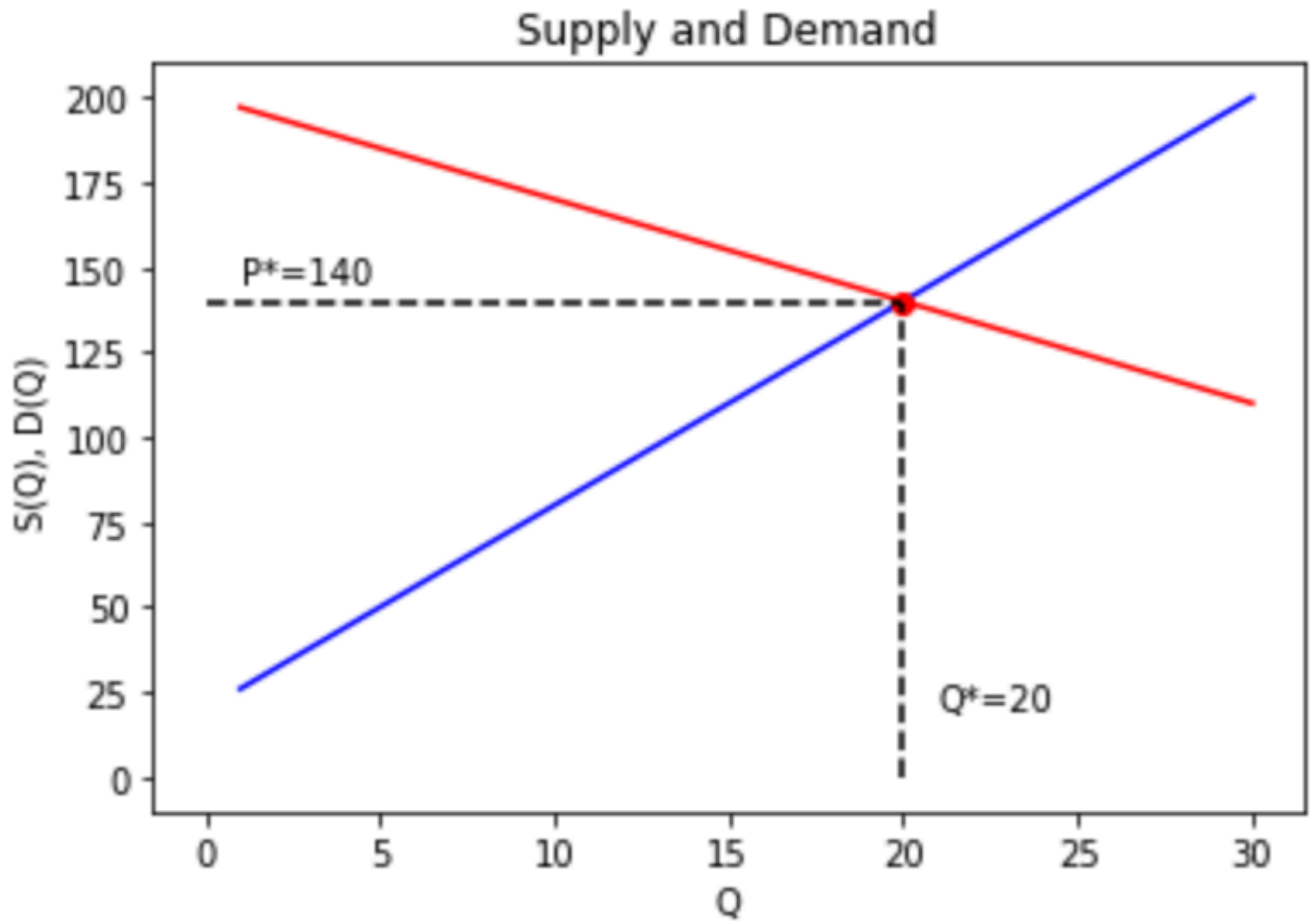
$P^* = 140$

$20 \times (140-20)/2$
$- 25$
$= 1175$

$20 \times (200 - 140)/2 = 600$

Check with the formulas !

MARKET EQUILIBRIUM — EXAMPLE



PRICE ELASTICITY

PRICE ELASTICITY OF DEMAND

$D(Q) = P$

$D^{-1}(P) = Q$

careful ! both functions are referred to as demand

- *Price elasticity* of "demand" is the variation in demand as a response to the variation in price. If the price increases, how would you expect the demand to change?

- $$E(p) = \left.\frac{dD^{-1}/D^{-1}}{dP/P}\right|_{P=p}$$

- $$E(p) = \frac{dD^{-1}(p)}{dP}\frac{P}{D^{-1}(p)}$$

- What is the elasticity of $D(Q) = 5 - Q$?

i) $D^{-1}(P) = 5 - P$

$dD^{-1}(P)/dP = -1$

$$E(p) = \frac{-P}{5-P}$$

PRICE ELASTICITY OF DEMAND

$$E(p) = \underbrace{\frac{dD^{-1}(p)}{dP}}_{\leq 0} \underbrace{\frac{p}{D^{-1}(p)}}_{> 0}$$

Elastic Demand

Inelastic Demand

Demand decreases at least as rapidly as price increases

Demand decreases slower than the price

PRICE ELASTICITY OF SUPPLY

$S(Q) = P$

$S^{-1}(P) = Q$

- *Price elasticity* of supply is the variation in supply as a response to the variation in price. If the price increases, how would you expect the supply to change?
- $$E(p) = \left.\frac{dS^{-1}/S^{-1}}{dP/P}\right|_{P=p}$$
- $$E(p) = \frac{dS^{-1}(p)}{dP}\frac{P}{S^{-1}(p)}$$
- What is the price elasticity of $S(Q) = 3 + 2Q$?

1) $S^{-1}(P) = \frac{P}{2} - \frac{3}{2}$

2) $dS^{-1}(P)/dP = 1/2$

3) $E(p) = \frac{P}{P-3}$

PRICE ELASTICITY OF SUPPLY

$$E(p) = \underbrace{\frac{dS^{-1}(p)}{dP}}_{\geq 0}\underbrace{\frac{P}{S^{-1}(p)}}_{\geq 0}$$

Inelastic Supply

Supply decreases slower than price

Elastic Supply

Supply decreases at least as rapidly as price increases

PRICE ELASTICITY — EXAMPLE

- What are the price elasticities of supply and demand at equilibrium in the market whose aggregate cost function is $C(Q) = 25 + 20Q + 3Q^2$ and the aggregate demand function is $D(Q) = 200 - Q$?

1) $S(Q) = 20 + 6Q = P \Rightarrow S^{-1}(P) = P/6 - 10/3$

$E_S(p) = \frac{P}{P - 20}$

2) $D(Q) = 200 - Q \Rightarrow D^{-1}(P) = 200 - P$

$E_D(p) = \frac{-P}{200 - P}$

IV - Government Interventions

WHY PREVENT MARKETS FROM BEING COMPETITIVE

- Governments can limit prices or quantities produced
- Welfare can be re-distributed through taxation

WHY PREVENT MARKETS FROM BEING COMPETITIVE

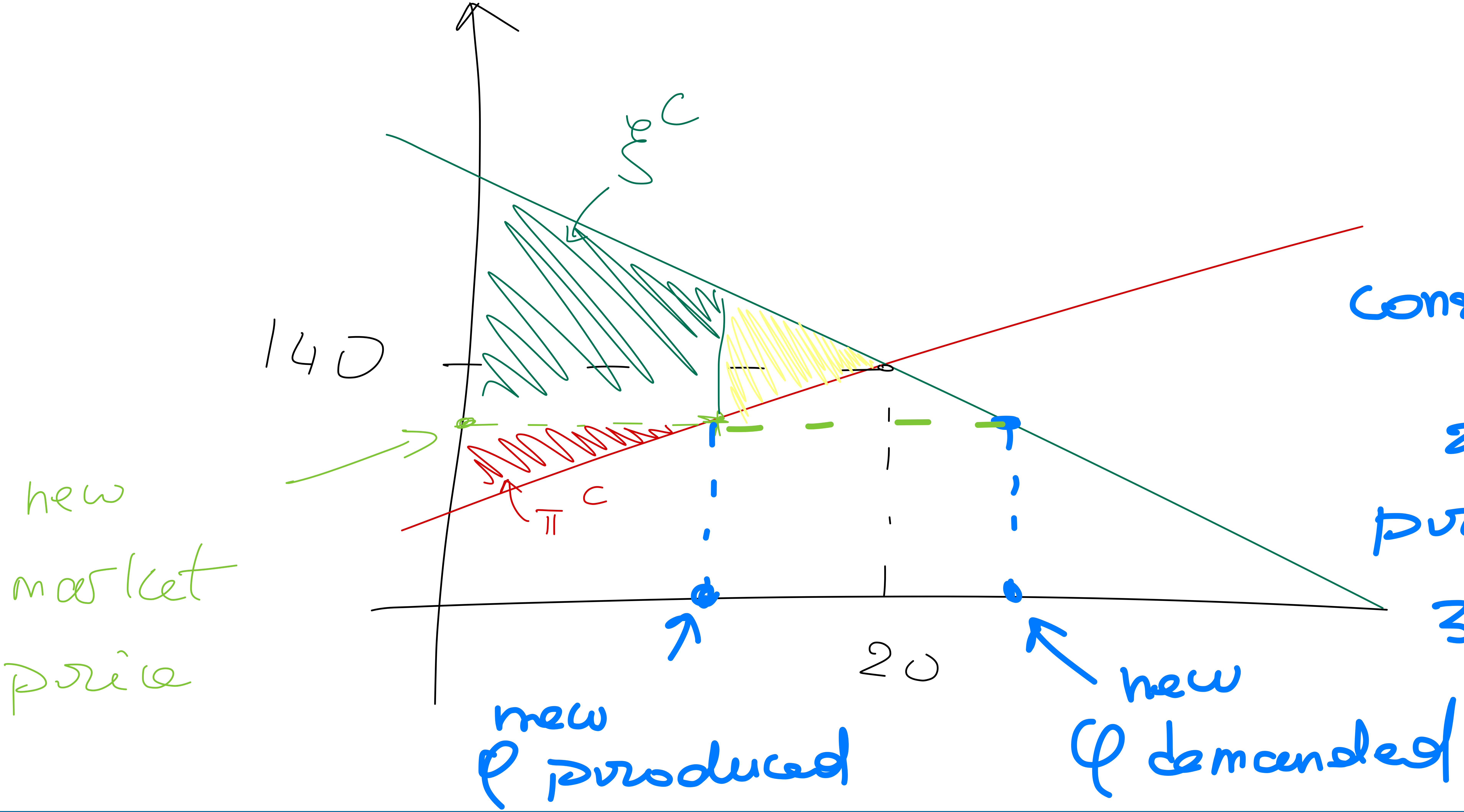
- Let's get back to our previous example in which $S(Q) = 20 + 6Q$ and $D(Q) = 200 - 3Q$. The market equilibrium is at $(Q^*, P^*) = (20,140)$.
- The producers' surplus was found to be $\Pi(Q^*) = \$1,175$, and the consumers' $\zeta(Q^*) = \$600$.

PRICE CEILING

PRICE CEILING

- Now, the consumers start protesting because of high prices, and the government decides to introduce a ***price ceiling*** of $128/car.
- Hence, the producers and now produce
  $$128 = 20 + 6Q' \implies Q' = 18$$
- What are the consumers and producers surplus?

PRICE CEILING



1) $\xi^c > \xi$
consumer surplus ↗

2) $\pi^c < \pi$
producer surplus ↘

3) Shortage

DEADWEIGHT LOSS

- ➤ How did the surpluses changed?

- ➤ The total welfare decrease is a reduction in the economy efficiency that occur when the competitive equilibrium is not achieved.

- ➤ This decrease is called ***Deadweight Loss***.

↳ It is the area

PRICE CEILING

- What happens to the quantity demanded by the consumers at the new price?

It increases, creating a production shortage.

PRICE FLOOR

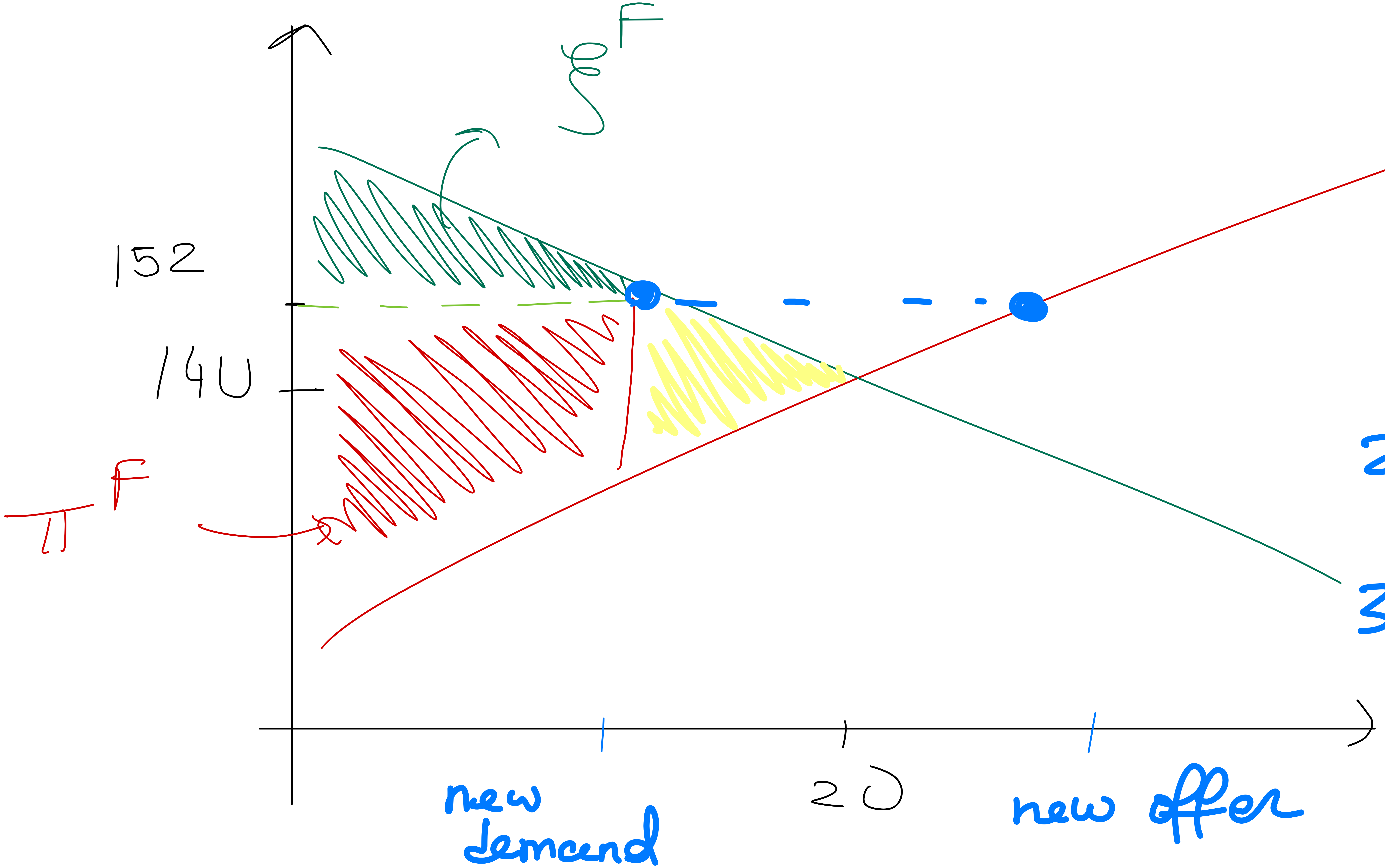
PRICE FLOOR

- Now, the producer threaten to relocate, and the government decides to introduce a ***price floor*** of \$152/ car.

- Hence, the producers and now produce
$$152 = 20 + 6Q' \implies Q' = 22$$

- What are the consumers and producers surplus?

PRICE FLOOR



1) $\xi^F < \xi$

2) $\pi^F > \pi$

3) surplus

DEADWEIGHT LOSS

- How did the surpluses changed?
- What happens to the quantity produced at the new price?

TAXATION

TAXATION

- The government taxes the producers' revenue at $t/unit.
- Consumers argue that the tax will impact the market price, hence leading to a diminishing consumer welfare.
- Producers argue that the tax will drive the costs up while they will need to maintain the prices low to keep selling, hence leading to a diminishing producer welfare.
- What do you think?

1) Consumers "buy" the product at

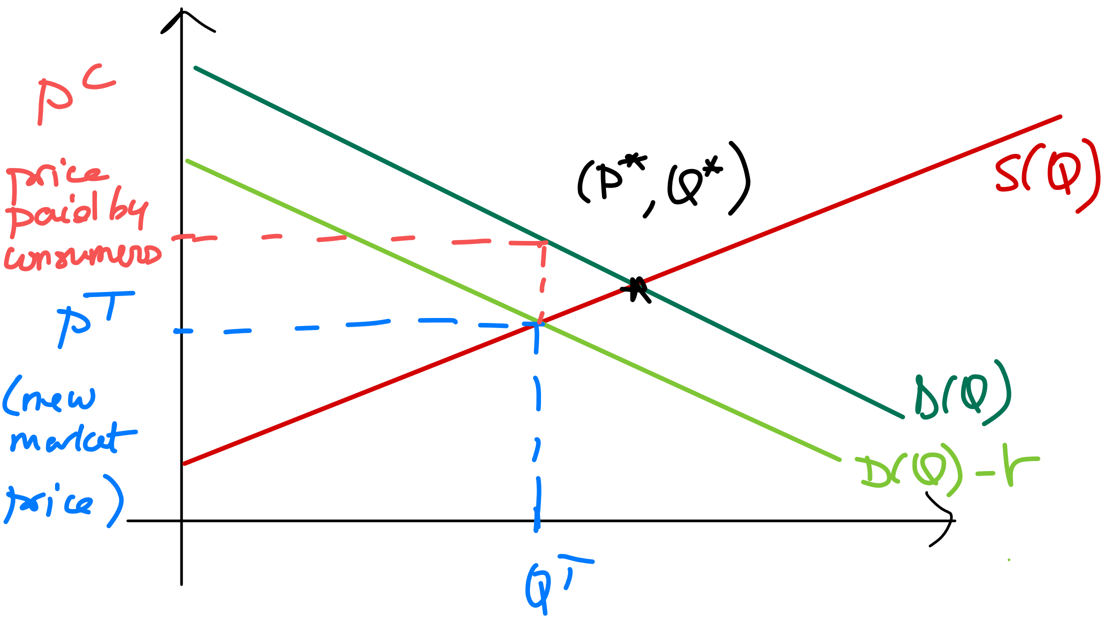
$$P^{tax} = P + t$$

Pre Tax : $S(Q^*) = D(Q^*) = P$

After Tax: (Q^T, P^T) s.t.

$$S(Q) = P$$

$$D(Q) = P + t = S(Q) + t$$



$$S(\varphi) = a\varphi + b$$

$$D(\varphi) = c\varphi + d$$

$$D(\varphi) - r = c\varphi + d - r$$

$$S(\varphi^*) = D(\varphi^*)$$

$$\Rightarrow \varphi^* = \frac{d-b}{a-c}$$

$$\boxed{p^* = \frac{ad-bc}{a-c}}$$

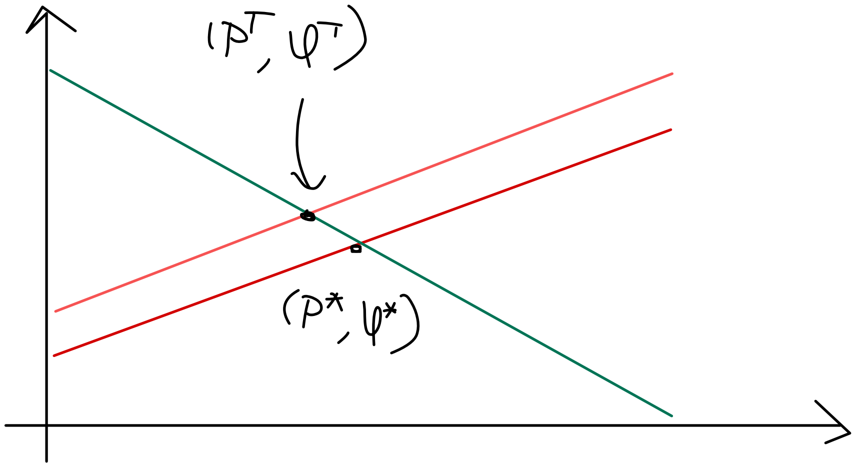
$$S(\varphi^T) = D(\varphi^T) - r$$

$$\Rightarrow \varphi^T = \frac{d-r-b}{a-c}$$

$$\boxed{p^T = \frac{ad-ar-bc}{a-c}}$$

$$\boxed{p^C = p^T + r}$$

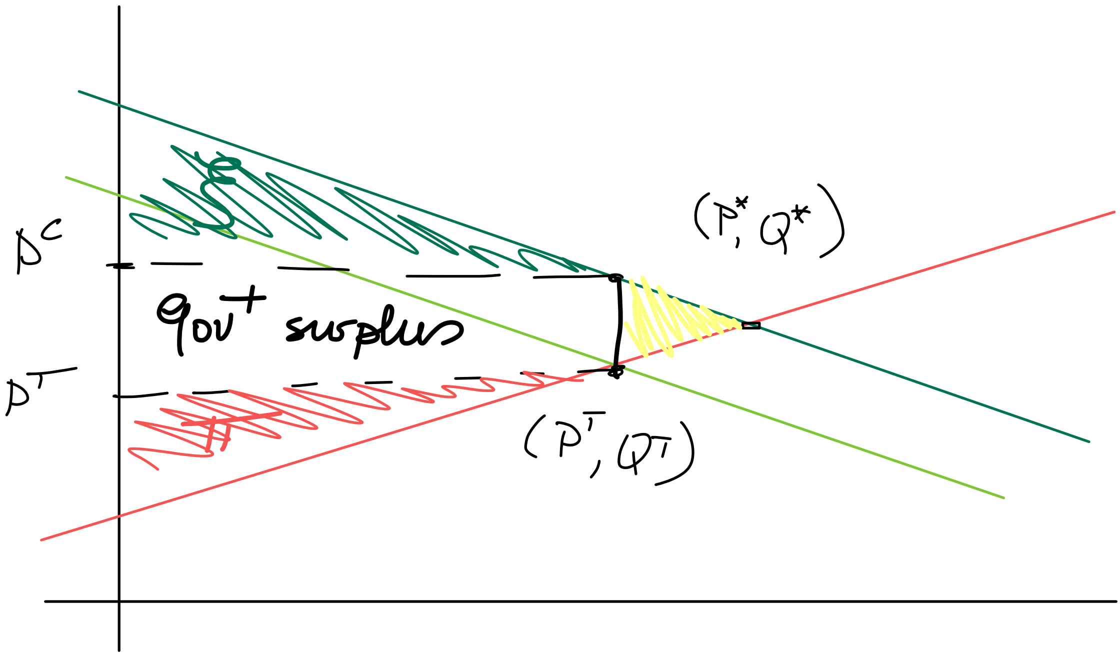
2) Producers only keep \$ $p - t$ / unit



After tax $S(Q) = P - t$

$D(Q) = P$ $\Rightarrow S(Q) + t = D(Q)$

same as before !



$$\xi^T = \frac{Q^T \times (d - P^C)}{2} \qquad \xi = \frac{Q^*(d - P^*)}{2}$$

$$\pi^T = \frac{Q^T \times (P^T - b)}{2} \qquad \pi = \frac{Q^*(P^* - b)}{2}$$

$$\varphi^* = \frac{d-b}{a-c} \qquad P^* = a\left(\frac{d-b}{a-c}\right) + b = \frac{ad-cb}{a-c}$$

$$\varphi^T = \frac{d-r-b}{a-c} = \varphi^* - \frac{r}{a-c} \qquad P^T = P^* - \frac{ar}{a-c} = P^c - r$$

$$\xi - \overline{\xi^T} = \frac{1}{2}\left(\varphi^*(d-P^*) - \varphi^T(d-P^c)\right) \qquad (1)$$

$$\pi - \overline{\pi^T} = \frac{1}{2}\left(\varphi^*(P^*-b) - \varphi^T(\overline{P^T}-b)\right) \qquad (2)$$

solve $(1) \geq (2)$!

...