IN DEFENSE OF FLUID DEMOCRACY

with Daniel Halpern, Joe Halpern, Ali Jadabaie, Elchanan Mossel and Ariel Procaccia

MANON REVEL
THE FUTURE OF DEMOCRACY

POLITICS WITHOUT POLITICIANS

The political scientist Helene Landemore asks, If government is for the people, why can't the people do the governing?

By Nathan Heller
February 19, 2020
ROAD MAP

* What is *fluid* democracy?
* Our fluid democracy model and *benchmarks* to evaluate its performance.
* *Scenarii* in which fluid democracy performs well (that is, better than direct democracy).
FLUID DEMOCRACY
WHAT IS FLUID DEMOCRACY?
WHY FLUID DEMOCRACY?
II

EPISTEMIC BENCHMARKS
THE EPISTEMIC APPROACH

➤ n agents vote on \{0, 1\} ground truth
➤ Person i votes according to \(X_i \sim \text{Ber}(p_i)\) where \(p_i \sim \mathcal{D}\)

➤ Power of aggregation of imperfect information: \(n\) (large enough) agents with \(p_i = .501\) vote better than one expert with \(p = .9999\)

★ Extended Condorcet's Jury Theorem (1785)

\[
\text{If } \mathbb{E}[\mathcal{D}] > \frac{1}{2}, \quad \lim_{n \to \infty} \mathbb{P}(\bar{X}_n > \frac{1}{2}) = 1
\]

\[
\bar{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i
\]


\[ \overline{X}_n = \frac{\sum_{i=1}^{n} X_i}{n} \quad \text{and} \quad FD = \frac{\sum_{i=1}^{n} w_i X_i}{n} \]

\[
\text{gain}(\overrightarrow{p}_n, G_n) = \mathbb{P}(FD > \frac{1}{2}) - \mathbb{P}(\overline{X}_n > \frac{1}{2})
\]

\[ G_n = (w_1, w_2, w_3, w_{>3}) \quad \text{and} \quad \overrightarrow{p}_n = (p_1, p_2, p_3, p_{>3}) \]
DELEGATION MECHANISM

\[ M = (q, \varphi) \]

\[
\begin{align*}
q & : [0,1] \rightarrow [0,1] \\
\varphi & : [0,1]^2 \rightarrow \mathbb{R}
\end{align*}
\]

- \( q(p_i) = \text{Probability that agent i delegates} \)
- \( \varphi(p_i, p_j) = \text{Weight agent i puts on agent j} \)
RECAP DEFINITIONS

- Delegation Instance \((\vec{p}_n, G_n)\)
- \(\text{gain}(\vec{p}_n, G_n) = \mathbb{P}(\text{FD} > \frac{1}{2}) - \mathbb{P}(\bar{X}_n > \frac{1}{2})\)

- Sampled \textbf{Competencies} \(\forall i \in [N], p_i \sim \mathcal{D}\)

- Sampled \textbf{Graph} through the Delegation Mechanism \(M = (q, \varphi)\)

- \(\text{gain}(\vec{p}_n, G_n)\) is hence a \textbf{Random Variable}
POSITIVE GAIN AND DO NO HARM

➤ There exists a distribution such that, the gain of fluid democracy is close to 1 for large enough instances, with high probability.

Definition (Probabilistic positive gain). A mechanism $M$ satisfies probabilistic positive gain with respect to a class $\mathcal{D}$ of distributions if there exists a distribution $\mathcal{D} \in \mathcal{D}$ such that for all $\varepsilon, \delta > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$,

$$\mathbb{P}_{\mathcal{D}, M, n}[\text{gain}(\bar{p}_n, G_n) \geq 1 - \varepsilon] > 1 - \delta.$$ 

➤ For all distributions, the loss of fluid democracy is arbitrarily small for large enough instances, with high probability.

Definition (Probabilistic do no harm). A mechanism $M$ satisfies probabilistic do no harm with respect to a class $\mathcal{D}$ of distributions if, for all distributions $\mathcal{D} \in \mathcal{D}$ and all $\varepsilon, \delta > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$,

$$\mathbb{P}_{\mathcal{D}, M, n}[\text{gain}(\bar{p}_n, G_n) \geq -\varepsilon] > 1 - \delta.$$
CORE LEMMA

Lemma

- Let $M$ a mechanism and $\mathcal{D}$ a class of distributions, if for all distribution in $\mathcal{D}$ there exists $\alpha$ such that

  (i) $\max\text{-weight}(G_n) = o(n)$ and (ii) $\sum_{i=1}^{n} w_i p_i / n - \sum_{i=1}^{n} p_i / n \geq 2\alpha$

  w.h.p., the mechanism satisfies probabilistic do no harm.

- Further, if there exists a distribution such that

  (iii) $\sum_{i=1}^{n} p_i / n \leq 1/2 - \alpha$ and $\sum_{i=1}^{n} w_i p_i / n \geq 1/2 + \alpha$

  w.h.p., the mechanism satisfies probabilistic positive gain.
We want to prove that w.h.p, $\text{gain}(\vec{p}_n, G_n) \geq -\varepsilon$

$\text{gain}(\vec{p}_n, G_n) \geq -\mathbb{P}(FD < \overline{X}_n)$

by the law of total probability

by (ii) $\sum_{i=1}^{n} w_i p_i - \sum_{i=1}^{n} p_i \geq 2an$

by Hoeffding Inequality

by (i) $\max\text{-weight}(G_n) = o(n)$

and Chebyshev Inequality

w.h.p. $\overline{X}_n \leq \sum_{i=1}^{n} w_i p_i$

$\geq FD$ w.h.p.
MECHANISMS
**Upward Delegation**

**Theorem 1**

For all $p \in (0, 1)$, the upward delegation mechanism $M = (q, \phi)$ such that $q(x) = p$ and $\phi(x, y) = 1_{\{y > x\}}$ satisfies probabilistic positive gain and do no harm with respect to the class of continuous distributions.

\[ \varphi(p_i, p_j) = 1_{\{p_j > p_i\}} \]

\[ q(p_i) = p \]
CORE LEMMA

Lemma

Let $M$ a mechanism and $\mathcal{D}$ a class of distributions, if for all distribution in $\mathcal{D}$ there exists $\alpha$ such that

(i) $\max\text{-weight}(G_n) = o(n)$ and (ii) \[ \sum_{i=1}^{n} w_i p_i/n - \sum_{i=1}^{n} p_i/n \geq 2\alpha \]

w.h.p., the mechanism satisfies probabilistic do no harm.

Further, if there exists a distribution such that

(iii) $\sum_{i=1}^{n} p_i/n \leq 1/2 - \alpha$ and $\sum_{i=1}^{n} w_i p_i/n \geq 1/2 + \alpha$ w.h.p., the mechanism satisfies probabilistic positive gain.
PROOF SKETCH
We want to show that $\mathbb{P} \left[ w \geq o(n) \right] \leq o(1)$.

By Markov Inequality, $\mathbb{P} \left[ w \geq o(n) \right] \leq \frac{\mathbb{E}[w]}{o(n)}$.

Some more work is actually needed to handle all the components.
PROOF SKETCH

\[ \sum_{i=1}^{n} w_i p_i / n - \sum_{i=1}^{n} p_i / n \geq 2\alpha \]

Condition (ii) is equivalent to saying that there is a positive displacement of expertise post-delegation.

A positive fraction of voters see their effective expertise increased by at least \((b - a)\). With high probability, the expertise post-delegation increased by \(p \pi_a \pi_b (b - a) / 8\).
PROOF SKETCH \[ \sum_{i=1}^{n} p_i/n \leq 1/2 - \alpha \] and \[ \sum_{i=1}^{n} w_i p_i/n \geq 1/2 + \alpha \]

For **Condition (iii)**, it suffices to choose a distribution of competence \( \mathcal{U}[0, 1 - 2\eta] \) with \( \eta \) small enough such that delegation pushes the average competence above a half.

0 \hspace{1cm} .25 - .5\eta \hspace{1cm} .5 - \eta \hspace{1cm} 1
Confidence Based

\[ \varphi(p_i, p_j) = 1 \]

**Theorem 2**

All confidence based mechanisms \( M = (q, \varphi) \) with monotonically decreasing \( q \) and \( \varphi(x, y) = 1 \) satisfy *probabilistic positive gain and do no harm* with respect to the class of continuous distributions.
\( \phi(p_i, p_j) \) increases in \( p_j \)

**General Continuous**

\[
\phi(p_i, p_j) \text{ increases in } p_j
\]

**Theorem 3**

For all \( p \in (0,1) \), all general continuous mechanisms \( M = (q, \varphi) \) with \( q(x) = p \) and \( \varphi \) is non-zero, continuous and increasing in its second coordinate satisfies *probabilistic positive gain and do no harm* with respect to the class of continuous distributions.

\[
q(p_i) = p
\]
Natural fluid democracy mechanisms are likely to lead to better voting results without the need for a central planner.

Performance of fluid democracy can be related to mild conditions on anti-concentration of power and an increase in the expected expertise at the heart of Condorcet's trade-off.

While these mechanisms rely on few assumptions, we do not have evidence that these are reasonable models.
FUTURE WORK

- Investigate reasonable mechanisms through a game-theoretic approach
- Discuss the new models of governance with political scientists and compare fluid democracy with sortition and proxy voting.
- Run real-life fluid democracy experiments at MIT!